On Some Nonlinear Effects of Heat Transport in Thermal Buffer Tubes

Konstantin Matveev, Scott Backhaus, and Greg Swift

Condensed Matter and Thermal Physics Group, Los Alamos National Laboratory,
Los Alamos, NM 87545

Abstract. Thermal buffer tubes and pulse tubes, elements of thermoacoustic systems, should thermally isolate their two ends at substantially different temperatures while acoustic power freely flows from end to end. In practical high-power operating regimes of thermoacoustic devices, large heat leaks due to acoustic streaming can appear along a thermal buffer tube, which may degrade the overall system performance. To study this effect, a controlled experimental system is under development that will allow measurements of the heat leak due to acoustic streaming under various acoustic and thermal conditions. The phenomenon under consideration has very rich physics, and some of the accompanying effects are discussed. Gravity may significantly reduce or augment streaming and heat transport, and a simple model has been derived to account for its effect. At the ends of a thermal buffer tube, the gas moves periodically between the nearly adiabatic environment of the tube and the nearly isothermal environment of the adjacent heat exchanger. This establishes specific temperature joining conditions at the tube ends, which are important for overall heat transport by acoustic streaming.

Keywords: Thermoacoustics, Acoustic Streaming, Thermal Buffer Tube, Joining Conditions.
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INTRODUCTION

Thermoacoustic engines and refrigerators either produce acoustic energy from, or pump thermal energy up, a mean temperature gradient [1]. They can be environmentally friendly, contain no moving mechanical parts, and have a great potential for applications where reliability is important, such as deep space travel and natural gas liquefaction on offshore platforms. At the present time, the low-amplitude regimes in these devices are well understood and can be modeled accurately, but finite-amplitude effects degrade our ability to accurately predict the performance in high-power regimes, which are of the most interest for practical applications.

One of these nonlinear effects is mass streaming, a second-order steady flow superimposed on and driven by the first-order acoustic oscillations. An arrangement typical for many thermoacoustic systems is shown in Fig. 1(a). Between the main ambient heat exchanger and the hot or cold heat exchanger lies a porous medium where the principal thermodynamic processes occur: entropy transport along the temperature gradient between heat exchangers and production or consumption of acoustic power. Many thermoacoustic systems also have a thermal buffer tube known as a “pulse tube” in pulse tube refrigerators) and a secondary ambient heat exchanger. Acoustic power is transmitted along this tube with minimal heat leak between the secondary ambient heat exchanger and the hot or cold heat exchanger. However, at high acoustic amplitudes, a second-order streaming flow appears that considerably increases the heat leak and results in significant degradation of the system performance. Rayleigh streaming, depicted in Fig. 1(a), is an internally circulating steady flow driven by boundary-layer interactions of thermoacoustic oscillations with the solid wall of the tube.

Significant progress in understanding acoustic streaming has been achieved in recent years. However, there are still many important open questions. Common analytical approaches usually assume that the streaming flow and its influence on the mean temperature field are small, and complicated solution procedures are described [2,3]. Under some assumptions, a practical recommendation based on analytical approaches can be made for preventing streaming, e.g., by tapering the tube [4]. Such small-streaming results are sometimes verified experimentally [5,6], but they may not be directly relevant to the high-power thermoacoustic systems where strong streaming is present [7,8]. CFD tools are promising for computing strong streaming flows [9-11], but significant problems, associated with multi-scale modeling in both space and time, remain. Also, specific boundary conditions appear at the ends of thermal buffer tubes [1,12]. The situation becomes even more difficult if other phenomena are accounted for, such as gravity, turbulence, and so on. There is a great need for a practical method to estimate streaming in thermoacoustic systems and for systematic and accurate experimental data on Rayleigh streaming in the presence of significant temperature gradients. Such a method and data series will be of great help to thermoacoustic practitioners. This paper describes our recent efforts in this direction.

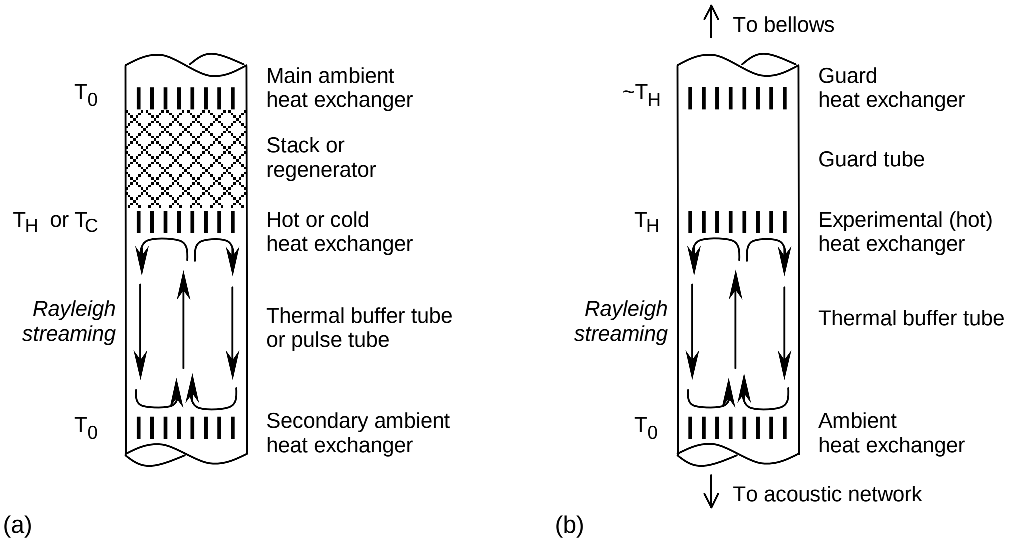

FIGURE 1. (a) System elements common to Stirling, pulse-tube, and stack-based thermoacoustic devices. Rayleigh streaming is shown inside a thermal buffer tube. (b) A part of the experimental system for measuring heat transport by Rayleigh streaming.

EXPERIMENTAL SYSTEM

A schematic of the main component of the experimental system, a vertically oriented TBT, is shown in Fig. 1(b). A linear-motor-driven bellows located above the tube produces acoustic oscillations, and an acoustic network attached to the bottom of the TBT allows us to regulate the ratio of the acoustic pressure and velocity amplitudes and the phase between them. The main goal of this system is to measure heat transfer by Rayleigh streaming from the experimental (hot) heat exchanger to the lower (ambient) heat exchanger. The heat transported by streaming in the TBT is the thermal power released at the experimental heat exchanger minus heat leaks by all non-TBT-streaming mechanisms. An additional guard tube section and a guard heat exchanger are placed above the experimental heat exchanger. The temperature of the guard heat exchanger is controlled to minimize heat transfer mechanisms in this part of the system, including streaming heat transfer in the guard tube.

The heat exchangers are copper cylinders with circular channels drilled through parallel to the direction of acoustic motion. Their temperature is set using either band heaters or aluminum blocks ported for circulating water. Flow straighteners, comprising a few layers of copper mesh, are employed at both ends of every heat exchanger to suppress jets produced by the heat exchanger channels.

At high amplitudes of the acoustic flow, significant nonlinear losses of acoustic power occur at the experimental heat exchanger, causing significant uncertainty in the thermal power released at this element. In order to minimize the heat added by acoustic dissipation, the heating element must be made as transparent as possible to the flow, though still providing effective heating. We have decided that a few sparse layers of nichrome wire weaved in a macor frame should be a good substitute for the heat exchanger currently employed, though we have not yet completed it.

To minimize heat leaks in the radial direction (to the environment), several layers of insulation cover the system, and a large-diameter copper pipe (shield) with distributed heaters and thermocouples is placed outside the insulation. By measuring temperatures on the experimental system components and comparing them with the local temperatures of the copper pipe at the same altitude, the shield is heated locally to minimize the temperature difference between the shield and system elements. Very thin walls on the TBT, just strong enough to sustain the high pressure inside the tube, minimize heat conduction along the tube walls.

From a methodological view, heat transfer by Rayleigh streaming should be measured as a function of various parameters including the difference in temperatures of the heat exchangers at the ends of the TBT. However, defining the end temperatures is not as trivial as in steady and/or incompressible flows. Gas parcels that cross the interface between the heat exchanger and the TBT during an acoustic cycle oscillate between nearly isothermal and nearly adiabatic environments, leading to complicated temperature joining conditions discussed below. To measure this temperature end effect, new parts were added to the system: a short tube section with a set of thermocouple wires stretched across the TBT, mounted between the TBT and the heat exchanger and used for determining mean gas temperatures far from the tube walls.

GRAVITY'S EFFECT ON STREAMING

Since the streaming in thermal buffer tubes occurs in the presence of a large temperature gradient, buoyancy can be important. Gravity can also play a significant role in predicting the behavior of thermoacoustic systems intended for the microgravity environment on spacecraft when such devices are developed and tested on Earth. We have derived a simple mathematical model to account for the influence of gravity on Rayleigh streaming and the associated convective heat transfer between two heat exchangers [13].

Assuming that the streaming is laminar and that the acoustic boundary layer thickness is small in comparison with the tube radius, the streaming flow in the TBT will have a pattern as shown in Fig. 2(a). The effective wall velocity U_w is computed from the streaming mass flux at the outer edge of the boundary layer [4]. This velocity depends on the first-order acoustic variables, temperature gradient, and gas properties. The variation of the flow velocity along the tube is assumed to be small in comparison with the variation of the flow velocity in the radial direction, the counterflowing streams are considered to have temperatures T_1 and T_2 for the inner and outer flows respectively, and the radial temperature variation within each of the two streams is ignored. With these assumptions, the appropriate momentum equation for the second-order (streaming) variables in each of the counterflowing streams is

$$-\frac{dp}{dx}+\frac{\mu}{R}\frac{d}{dR}\left(R\frac{dU}{dR}\right)-\rho g=0\ , \tag{1}$$

where density ρ and viscosity μ are different for the two streams because they depend on temperature.

The relevant dimensionless parameters in this problem are the following:

$$G=\frac{\rho_1 g R_t^2}{\mu_1 U_w},\qquad \alpha=\frac{\mu_1}{\mu_2},\qquad \beta=\frac{\rho_2}{\rho_1}, \tag{2}$$

where the first parameter characterizes the importance of the buoyancy with respect to viscous shear (index 1 indicates the inner stream), and the other parameters are the ratios of viscosities and densities in the two counterflowing streams. An implicit analytical solution for the velocity profiles has been obtained [13]. Examples of dimensionless velocity profiles across the tube are given in Fig. 2(b) for one value of G and three values of the difference in the stream temperatures. The driving wall velocity is directed upward, gravity is oriented downward, and the hot heat exchanger is above the cold heat exchanger. In the isothermal case, the profile is parabolic [dashed line in Fig. 2(b)]. As the temperature difference rises, the streaming-reducing buoyancy increases. A similar effect is produced by the magnitude of G: the higher this number, the stronger the suppression of the streaming by gravity.

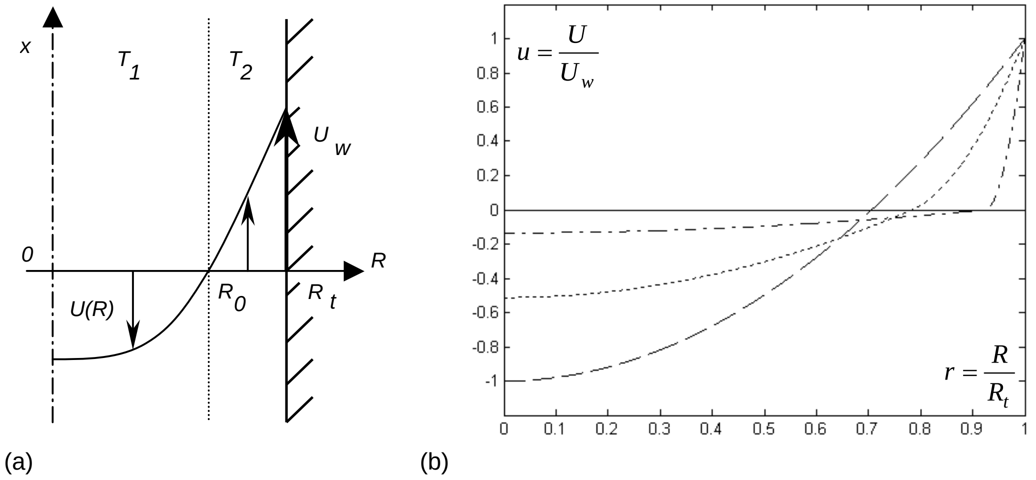

FIGURE 2. (a) A magnified view of a region inside the tube. The mean flow pattern *U(R)* is caused by Rayleigh streaming. (b) Streaming velocity profiles at $G = 2500$. Dashed line, $\alpha = \beta = 1$; dotted line, $\alpha \approx \beta \approx 1.01$; dash-dotted line, $\alpha \approx \beta \approx 1.1$.

This approach can be extended for approximate estimations of the effect of gravity on streaming. Practical analysis can be organized in an iterative procedure as follows. First, the temperature field inside the tube can be guessed; for example, as in the absence of acoustic oscillations. Then, assuming the acoustic pressure does not vary across the tube, the acoustic wave equation can be solved for the first-order acoustic variables. The boundary-layer mass flux that drives streaming can then be found along the tube wall [4]. This mass flux determines the effective wall velocity. Using a solution of the steady Navier-Stokes equation (e.g., from a suitable CFD tool), the second-order steady streaming flow and the temperature field inside the tube can be calculated, applying appropriate boundary conditions on the tube walls and at the tube ends. The boundary-layer entropy flow and the heat conduction through the tube walls can be included. After that, one can return to the wave equation step, and carry on iterations until a converged solution for the streaming flow and mean temperature distribution is found. Then, the heat leaks by both mass streaming and the boundary-layer entropy flow can be calculated.

TEMPERATURE JOINING CONDITIONS

The thermal boundary conditions at the tube ends for the streaming problem are complicated due to oscillating motion of gas parcels through the interface between nearly isothermal and adiabatic regions. A simple explanation for this effect is given in [1] and more detailed experimental and modeling study is presented in [12].

Some understanding of this phenomenon can be obtained from the idealized temperature–position trajectories of the gas parcels in the vicinity of the heat exchanger, as shown in Fig. 3. The heat exchanger has a fixed temperature T_{HX}, and this space is nearly isothermal. The space in the TBT is nearly adiabatic. The characteristic length in this problem is a peak-to-peak acoustic displacement. Assuming one-dimensional flow, Fig. 3(b) shows the trajectories of the gas parcels on a temperature–position diagram for the case of acoustic pressure in phase with acoustic velocity, which corresponds to a traveling wave. Pressure and temperature oscillations lead the gas parcel displacement by 90^{o}, and the result is an elliptical trajectory for the "particular" gas parcel [shown by a bold ellipse in Fig. 3 b)] that just touches the heat exchanger at its leftmost position. Parcels to the right of the particular parcel follow similar elliptic trajectories, while parcels to the left follow truncated ellipses with a temperature of T_{HX} while inside the heat exchanger. It is interesting that these gas parcels return to the heat exchanger with temperatures different from T_{HX}. The mean temperature profile inside the peak-to-peak zone acquires a curved shape, illustrating the complexity of defining the thermal boundary conditions at the tube ends. The mean temperature profile in the gas at the tube ends depends on the acoustic pressure and displacement amplitudes, the phase between them, the temperature of the heat exchanger, the temperature gradient in the TBT beyond the peak-to-peak zone, and gas properties.

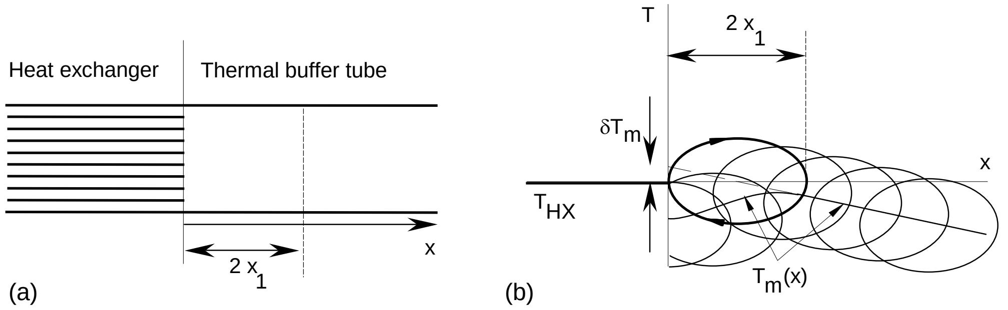

FIGURE 3. (a) An interface between an ideal heat exchanger and a thermal buffer tube. (b) Temperature-position diagram of the gas parcels with traveling-wave phasing. Adapted from [1].

A mathematical model that employs a Lagrangian-based method to track gas parcels in space and time has been developed for predicting the temperature profile. The mean temperature at a certain location is found by averaging temperatures of the gas parcels crossing this location over an acoustic cycle. Though the molecular heat conduction plays a minor role in the temperature end effect, the effective heat conduction can be augmented by vortices shed from the flow straighteners that are installed at the interface between the heat exchanger and the TBT. Modeling was carried out for both zero conduction and enhanced heat conduction.

Using the short TBT section described earlier, the temperature profile at the TBT ends was measured. An example of experimental data and model results for the mean gas temperature profiles in the TBT close to its interface with the heat exchanger is shown in Fig. 4 for traveling-wave phasing. Depending on the traveling-wave direction, the temperatures of gas parcels either drop or rise when they leave the heat exchanger, leading to the appearance of a dip or bump in mean temperature profiles, noticeable in Fig. 4. This deviation ends at the peak-to-peak displacement. The model that neglects heat conduction qualitatively predicts the temperature deviation, but the magnitude of this effect is exaggerated. The enhanced heat conduction smoothes the temperature deviation, giving results in good quantitative agreement with the test data at traveling-wave phasing.

When the phase of the acoustic wave was significantly different from 0^o or 180^o (e.g., by 60^o), some discrepancy developed between the theory and the experiment. Unfortunately, in the present experimental system configuration, it is not possible to closely approach the standing wave phasing, so we cannot conclude how well the model predicts the temperature joining conditions in a standing wave.

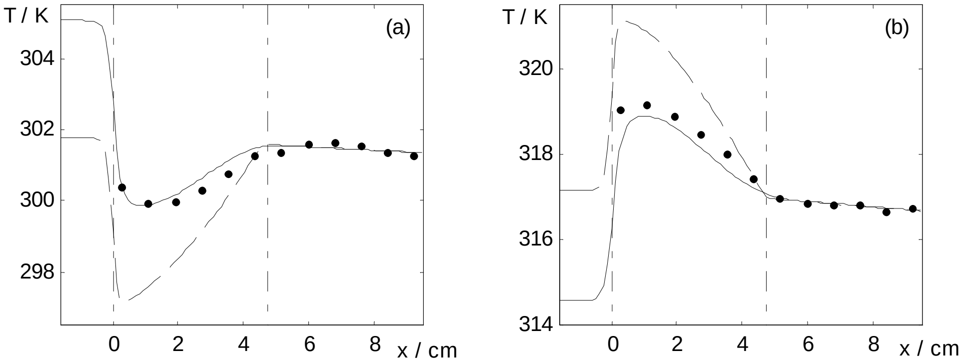

FIGURE 4. Mean temperatures of the gas (helium) at the TBT ends in the traveling wave. Points, experimental data; solid line, calculations with enhanced heat conduction; dashed line, calculations with zero heat conduction. Vertical dash-dotted line at $x = 0$ corresponds to the interface between the heat exchanger and the TBT; the other vertical dash-dotted line is located at one peak-to-peak displacement from the heat exchanger. Direction of the acoustic wave propagation: (a) from heat exchanger to TBT; (b) from TBT to heat exchanger.

CONCLUDING REMARKS

Higher performance and robustness are required from the next-generation high-power thermoacoustic devices in order to achieve their successful commercialization. However, nonlinear effects arising in high-amplitude regimes of operation of thermoacoustic systems are not yet sufficiently understood. From a scientific standpoint, these nonlinear phenomena also represent very interesting research challenges. Mass streaming, which is induced by first-order acoustic oscillations, has significant consequences for heat transfer in thermoacoustic devices and can be either harmful or useful. Turbulence in oscillating flows, stability and hysteresis of streaming patterns, and variable system orientation with respect to the gravity are problems waiting for future research.

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